**Turma B:**

a) Decompondo em frações parciais obtemos $\frac{4x^2+7x+26}{x(x^2+6x+13)} = \frac{2}{x} + \frac{2x-5}{x^2+6x+13}$. Logo

$$\begin{aligned}
\int \frac{4x^2+7x+26}{x^3+6x^2+13x}dx &= \int \frac{4x^2+7x+26}{x(x^2+6x+13)}dx \\
&= \int \frac{2}{x}dx + \int \frac{2x-5}{x^2+6x+13}dx \\
&= \ln(x^2) + \int \frac{2x-5}{x^2+6x+13}dx \\
&= \ln(x^2) + \int \frac{2x+6}{x^2+6x+13}dx - \int \frac{11}{x^2+6x+13}dx \\
&= \ln(x^2) + \ln(x^2+6x+13) - 11\int \frac{1}{4+(x+3)^2}dx
\end{aligned}$$

A última integral é resolvida via substituição $x+3 = 2\tan(\theta)$

$$\int \frac{1}{4+(x+3)^2}dx = \frac{1}{2}\int d\theta = \frac{1}{2}\arctan(\frac{x+3}{2}) + c$$

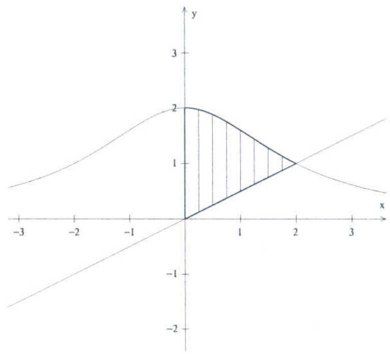
A solução é então

$$\int \frac{4x^2+7x+26}{x^3+6x^2+13x}dx = \ln(x^2) + \ln(x^2+6x+13) - \frac{11}{2}\arctan(\frac{x+3}{2}) + c$$

b) Integrando por partes com $f = \ln(\cos(x))$ e $g' = \sec^2(x)$ temos

$$\begin{aligned}
\int \sec^2(x)\ln(\cos(x))dx &= \ln(\cos(x))\tan(x) + \int \tan^2(x)dx \\
&= \ln(\cos(x))\tan(x) + \int (\sec^2(x)-1)dx \\
&= \ln(\cos(x))\tan(x) + \tan(x) - x + c
\end{aligned}$$

**Questão 2.** Seja $R$ a região compreendida entre os gráficos de $f(x) = \dfrac{8}{4+x^2}$ e $g(x) = \dfrac{x}{2}$ para $x \in [0,2]$, como mostra a figura abaixo:




(2,0) a) Calcule o volume do sólido obtido pela rotação de $R$ em torno do eixo $Ox$.

(1,5) b) Calcule o volume do sólido obtido pela rotação de $R$ em torno do eixo $Oy$.

a) $V_x = V_1 - V_2$ , $V_1 = \int_0^2 \pi \left(\frac{8}{4+x^2}\right)^2 dx$ , $V_2 = \int_0^2 \pi \left(\frac{x}{2}\right)^2 dx$

$$V_1 = 64\pi \int_0^2 \frac{1}{(4+x^2)^2}\,dx \underset{x=2\,tg\,t}{=} 64\pi \int_0^{\pi/4} \frac{2\sec^2 t\,dt}{(4+4\,tg^2 t)^2} =$$

$$= 8\pi \int_0^{\pi/4} \frac{1}{\sec^2 t}\,dt = 8\pi \int_0^{\pi/4} \cos^2 t\,dt =$$

$$= 8\pi \int_0^{\pi/4} \left(\frac{1}{2} + \frac{1}{2}\cos(2t)\right) dt =$$

$$= 8\pi \left[\frac{1}{2}\frac{\pi}{4} + \frac{1}{2}\cdot\frac{1}{2}\,\mathrm{sen}(2t)\Big|_0^{\pi/4}\right] = 8\pi\left(\frac{\pi}{8} + \frac{1}{4}\right)$$

$$= \pi^2 + 2\pi$$

$$V_2 = \frac{\pi}{4}\,\frac{x^3}{3}\Big|_0^2 = \frac{2\pi}{3}$$

$$V_x = \pi^2 + 2\pi - \frac{2\pi}{3} = \pi^2 + \frac{4\pi}{3}$$

b) $y = \frac{x}{2} \Leftrightarrow x = 2y$

$y = \frac{8}{4+x^2}$ , $x \geq 0$ $\Leftrightarrow \begin{cases} 4+x^2 = \frac{8}{y} \\ x \geq 0 \end{cases} \Leftrightarrow x = \sqrt{\frac{8}{y} - 4}$

$V_y = V_3 + V_4$

$V_3 = \int_0^1 \pi (2y)^2 dy = \pi \cdot 4 \frac{y^3}{3} \Big|_0^1 = \frac{4}{3}\pi$

$V_4 = \int_1^2 \pi \left(\sqrt{\frac{8}{y} - 4}\right)^2 dy = \pi \int_1^2 \left(\frac{8}{y} - 4\right) dy =$

$= \pi \left[ 8 \ln y \Big|_1^2 - 4y \Big|_1^2 \right] = \pi (8 \ln 2 - 4)$

$V_y = V_3 + V_4 = \pi \left(8 \ln 2 - 4 + \frac{4}{3}\right) = \pi \left(8 \ln 2 - \frac{2}{3}\right)$

**Questão 3.**

I) Sejam $f(x) = \sqrt[5]{x}$ e $x_0 \neq 0$.

(1,0) a) Determine o polinômio de Taylor de ordem 3 de $f$ em volta de $x_0$.

$$p_3(x) = x_0^{\frac{1}{5}} + \tfrac{1}{5}x_0^{-\frac{4}{5}}(x-x_0) - \tfrac{4}{5^2.2}x_0^{-\frac{9}{5}}(x-x_0)^2 + \tfrac{36}{5^3.3!}x_0^{-\frac{14}{5}}(x-x_0)^3$$

$$E(x) = \tfrac{f^{(4)}(\bar{x})}{4!}(x-x_0)^4 = -\tfrac{504}{5^4.4!}\bar{x}^{-\frac{19}{5}}(x-x_0)^4 = -\tfrac{21}{5^4}\bar{x}^{-\frac{19}{5}}(x-x_0)^4$$

(1,0) b) Usando o item (a) para $x_0 = 32$, encontre um valor aproximado para $\sqrt[5]{34}$ e decida se o erro, em módulo, é inferior a $\dfrac{1}{5^2 \cdot 2^{15}}$.

$$p_3(x) = 2 + \tfrac{1}{5.2^4}(x-32) - \tfrac{1}{5^2.2^8}(x-32)^2 + \tfrac{3}{5^3.2^{13}}(x-32)^3$$

$$\sqrt[5]{34} \approx p_3(34) = 2 + \tfrac{1}{5.2^4}(34-32) - \tfrac{1}{5^2.2^8}(34-32)^2 + \tfrac{3}{5^3.2^{13}}(34-32)^3$$

$$\sqrt[5]{34} \approx 2 + \tfrac{1}{40} - \tfrac{1}{1600} + \tfrac{3}{128000} = 2 + 0,025 - 0,000625 + 0,0000234 = 2,0243984.$$

$$E(x) = -\frac{21}{5^4\bar{x}^{\frac{19}{5}}} \cdot (x-32)^4 \Rightarrow E(34) = -\frac{21}{5^4\bar{x}^{\frac{19}{5}}} \cdot (34-32)^4 = -\frac{21}{25.5^2\bar{x}^{\frac{19}{5}}} \cdot 2^4$$

Como $\bar{x} > 32$, pois $32 < \bar{x} < 34$, temos que $\frac{1}{\bar{x}} < \frac{1}{32} = \frac{1}{2^5}$.

Portanto, $|E(34)| = |-\frac{21}{25.5^2\bar{x}^{\frac{19}{5}}} \cdot 2^4| = \frac{21}{25} \cdot \frac{1}{5^2} \cdot \frac{1}{\bar{x}^{\frac{19}{5}}} \cdot 2^4 < \frac{21}{25} \cdot \frac{1}{5^2} \cdot \frac{2^4}{2^{19}} < \frac{1}{5^2.2^{15}}$.

(1,5) II) Seja $F(x) = \displaystyle\int_{\cos x}^{\operatorname{sen} x} e^{4x+t^2}\,dt$. Calcule $F'(x)$ e determine $F'(\frac{\pi}{4})$.

$$F'(x) = 4e^{4x}.\int_{\cos x}^{\operatorname{sen} x} e^{t^2}\,dt + e^{4x}[e^{\operatorname{sen}^2 x}.\cos x - e^{\cos^2 x}.(-\operatorname{sen} x)]$$

$$F'(\tfrac{\pi}{4}) = 4.0 + e^{\pi}\left[e^{\frac{1}{2}}.\frac{\sqrt{2}}{2} - e^{\frac{1}{2}}.(-\frac{\sqrt{2}}{2})\right] = e^{\pi+\frac{1}{2}}\sqrt{2}$$