Questão 1

Turma A:

a) Decompondo em frações parciais obtemos $\dfrac{4x^2+13x+16}{x(x^2+4x+8)} = \dfrac{2}{x} + \dfrac{2x+5}{x^2+4x+8}$. Logo

$$\begin{aligned}
\int \frac{4x^2+13x+16}{x^3+4x^2+8x}dx &= \int \frac{4x^2+13x+16}{x(x^2+4x+8)}dx \\
&= \int \frac{2}{x}dx + \int \frac{2x+5}{x^2+4x+8}dx \\
&= \ln(x^2) + \int \frac{2x+5}{x^2+4x+8}dx \\
&= \ln(x^2) + \int \frac{2x+4}{x^2+4x+8}dx + \int \frac{1}{x^2+4x+8}dx \\
&= \ln(x^2) + \ln(x^2+4x+8) + \int \frac{1}{4+(x+2)^2}dx
\end{aligned}$$

A última integral é resolvida via substituição $x+2 = 2\tan(\theta)$

$$\int \frac{1}{4+(x+2)^2}dx = \frac{1}{2}\int d\theta = \frac{1}{2}\arctan(\frac{x+2}{2}) + c$$

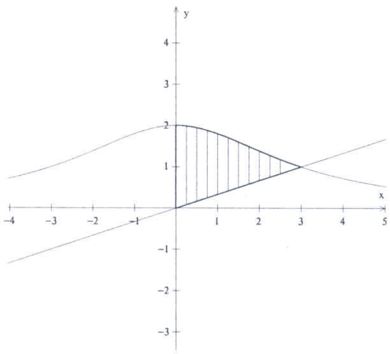
A solução é então

$$\int \frac{4x^2+13x+16}{x^3+4x^2+8x}dx = \ln(x^2) + \ln(x^2+4x+8) + \frac{1}{2}\arctan(\frac{x+2}{2}) + c$$

b) Integrando por partes com $f = \ln(\cos(x))$ e $g' = \sec^2(x)$ temos

$$\begin{aligned}
\int \sec^2(x)\ln(\cos(x))dx &= \ln(\cos(x))\tan(x) + \int \tan^2(x)dx \\
&= \ln(\cos(x))\tan(x) + \int (\sec^2(x) - 1)dx \\
&= \ln(\cos(x))\tan(x) + \tan(x) - x + c
\end{aligned}$$

Questão 2. Seja R a região compreendida entre os gráficos de $f(x) = \dfrac{18}{9+x^2}$ e $g(x) = \dfrac{x}{3}$ para $x \in [0,3]$, como mostra a figura abaixo:



(2,0) a) Calcule o volume do sólido obtido pela rotação de R em torno do eixo Ox.

(1,5) b) Calcule o volume do sólido obtido pela rotação de R em torno do eixo Oy.

a) $V_x = V_1 - V_2$

$$V_1 = \int_0^3 \pi \left(\frac{18}{9+x^2}\right)^2 dx = 18^2 \pi \int_0^3 \frac{1}{(9+x^2)^2} dx \underset{x = 3\,tg\,t}{=}$$

$$= 18^2 \pi \int_0^{\pi/4} \frac{3 \sec^2 t}{(9+9tg^2 t)^2} dt = 12\pi \int_0^{\pi/4} \frac{1}{\sec^2 t} dt =$$

$$= 12\pi \int_0^{\pi/4} \cos^2 t \, dt = 12\pi \int_0^{\pi/4} \left(\frac{1}{2} + \frac{1}{2}\cos(2t)\right) dt =$$

$$= 12\pi \left[\frac{1}{2}\frac{\pi}{4} + \frac{1}{2}\frac{sen(2t)}{2}\Big|_0^{\pi/4}\right] = 12\pi\left(\frac{\pi}{8} + \frac{1}{4}\right)$$

$$= \frac{3}{2}\pi^2 + 3\pi$$

$$V_2 = \int_0^3 \pi \left(\frac{x}{3}\right)^2 dx = \frac{\pi}{9}\frac{x^3}{3}\Big|_0^3 = \pi$$

$$V_x = \frac{3}{2}\pi^2 + 3\pi - \pi = \frac{3}{2}\pi^2 + 2\pi$$

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b) $y = \frac{x}{3} \iff x = 3y$

$y = \frac{18}{9+x^2}, x \geq 0 \iff 9 + x^2 = \frac{18}{y} \quad x \geq 0 \iff$

$x = \sqrt{\frac{18}{y} - 9}$

$V_y = V_3 + V_4$

$V_3 = \int_0^1 \pi (3y)^2 dy = 9\pi \frac{y^3}{3} \Big|_0^1 = 3\pi$

$V_4 = \int_1^2 \pi \left(\sqrt{\frac{18}{y} - 9} \right)^2 dy = \pi \int_1^2 \left(\frac{18}{y} - 9 \right) dy =$

$= \pi \left(18 \ln y \Big|_1^2 - 9y \Big|_1^2 \right) = \pi \left(18 \ln 2 - 9 \right)$

$V_y = 3\pi + \pi(18 \ln 2 - 9) = \pi(18 \ln 2 - 6) = 6\pi(3\ln 2 - 1$

Questão 3.

I) Sejam $f(x) = \sqrt[5]{x}$ e $x_0 \neq 0$.

$$p_3(x) = {x_0}^{\frac{1}{5}} + \tfrac{1}{5}{x_0}^{-\frac{4}{5}}(x - x_0) - \tfrac{4}{5^2.2}{x_0}^{-\frac{9}{5}}(x - x_0)^2 + \tfrac{36}{5^3.3!}{x_0}^{-\frac{14}{5}}(x - x_0)^3$$

$$E(x) = -\tfrac{21}{5^4}\bar{x}^{-\frac{19}{5}}(x - x_0)^4$$

(1,0) b) Usando o item (a) para $x_0 = 32$, encontre um valor aproximado para $\sqrt[5]{34}$ e decida se o erro, em módulo, é inferior a $\dfrac{1}{5^2 \cdot 2^{15}}$.

$$p_3(x) = 2 + \tfrac{1}{5.2^4}(x - 32) - \tfrac{1}{5^2.2^8}(x - 32)^2 + \tfrac{3}{5^3.2^{13}}(x - 32)^3$$

$$\sqrt[5]{34} \approx p_3(34) = 2 + \tfrac{1}{5.2^4}(34 - 32) - \tfrac{1}{5^2.2^8}(34 - 32)^2 + \tfrac{3}{5^3.2^{13}}(34 - 32)^3$$

$$\sqrt[5]{34} \approx 2 + \tfrac{1}{40} - \tfrac{1}{1600} + \tfrac{3}{128000} = 2 + 0,025 - 0,000625 + 0,0000234 = 2,0243984.$$

$$E(x) = -\frac{21}{5^4\bar{x}^{\frac{19}{5}}} \cdot (x - 32)^4 \Rightarrow E(34) = -\frac{21}{5^4\bar{x}^{\frac{19}{5}}} \cdot (34 - 32)^4 = -\frac{21}{25.5^2\bar{x}^{\frac{19}{5}}} \cdot 2^4$$

Como $\bar{x} > 32$, pois $32 < \bar{x} < 34$, temos que $\frac{1}{\bar{x}} < \frac{1}{32} = \frac{1}{2^5}$.

Portanto, $|E(34)| = |-\frac{21}{25.5^2\bar{x}^{\frac{19}{5}}} \cdot 2^4| = \frac{21}{25} \cdot \frac{1}{5^2} \cdot \frac{1}{\bar{x}^{\frac{19}{5}}} \cdot 2^4 < \frac{21}{25} \cdot \frac{1}{5^2} \cdot \frac{2^4}{2^{19}} < \frac{1}{5^2.2^{15}}$.

(1,5) II) Seja $F(x) = \displaystyle\int_{\operatorname{sen} x}^{\cos x} e^{2x+t^2}\, dt$. Calcule $F'(x)$ e determine $F'(\frac{\pi}{4})$.

$$F'(x) = 2e^{2x}.\int_{\operatorname{sen} x}^{\cos x} e^{t^2}\, dt + e^{2x}[e^{\cos^2 x}.(-\operatorname{sen} x) - e^{\operatorname{sen}^2 x}.\cos x]$$

$$F'(\tfrac{\pi}{4}) = 2.0 + e^{\frac{\pi}{2}}\left[e^{\frac{1}{2}}.(-\frac{\sqrt{2}}{2}) - e^{\frac{1}{2}}.\frac{\sqrt{2}}{2}\right] = -e^{\frac{\pi}{2}+\frac{1}{2}}\sqrt{2}$$