

18.01 Final Exam

Name: _____

Please circle the number of your recitation.

1. 10am Tyomkin	Problem 1: _____ /25	Problem 6: _____ /25
2. 10am Kilic	Problem 2: _____ /25	Problem 7: _____ /25
3. 12pm Coskun	Problem 3: _____ /25	Problem 8: _____ /25
4. 1pm Coskun	Problem 4: _____ /25	Problem 9: _____ /25
5. 2pm Hancock	Problem 5: _____ /25	Problem 10: _____ /25
6. 2pm Tyomkin		
7. 2pm Starr		
8. 3pm Hancock		
		Total: _____ /250

Instructions: Please write your name at the top of every page of the exam. The exam is closed book, calculators are not allowed, but you are allowed to use your prepared index card. You will have approximately 3 hours for this exam. The point value of each problem is written next to the problem – use your time wisely. Please show all work, unless instructed otherwise. Partial credit will be given only for work shown.

You may use either pencil or ink. If you have a question, need extra paper, need to use the restroom, etc., raise your hand.

Name: _____

Problem 1: _____ /25

Problem 1(25 points) Use the definition of the derivative as a limit of difference quotients to compute the derivative of,

$$f(x) = e^x,$$

for all real numbers x . You may use the fact that the following limit exists and has the given value,

$$\lim_{h \rightarrow 0} \frac{e^h - 1}{h} = 1.$$

Show all work.

Name: _____

Problem 2: _____ /25

Problem 2(25 points) On the graph provided, sketch the graph of the function,

$$y = \frac{x^2 - 2}{x^2 - 1}.$$

In particular, label all vertical asymptotes, all horizontal asymptotes, all local maxima, all local minima, and all inflection points. On the first number line provided, mark where y'' is positive, where y'' is negative, and where y'' is zero. On the second number line provided, mark where y' is positive, where y' is negative, and where y' is zero. Above the third number line, indicate whether y is increasing, where y is decreasing, and where y has a critical point. Below the third number line, indicate where y is concave up, where y is concave down, and where y has an inflection point.

Name: _____

Problem 2, continued

Name: _____

Problem 3: _____ /25

Problem 3(25 points) An aerosol can is designed as in the figure. The sides of the can form a cylinder of base radius r and height h . The top of the can is a hemisphere of radius r pointing up, and the bottom of the can is also a hemisphere of radius r pointing up. The volume of a cylinder of base radius r and height h is $\pi r^2 h$, and the area of the sides of the cylinder is $2\pi r h$. The volume of a sphere of radius r is $(4/3)\pi r^3$, and the surface area is $4\pi r^2$. What ratio of height h to radius r maximizes the volume of the can for a fixed surface area? Show all work.

Name: _____

Problem 3, continued

Name: _____

Problem 4: _____ /25

Problem 4(25 points) Let $f(x)$ be a Riemann integrable function defined on the interval $[0, 1]$.

(a)(5 points) Write down the general formula for the Riemann sum approximating the Riemann integral,

$$\int_0^1 f(x)dx,$$

for the partition of $[0, 1]$ into n subintervals of equal length. Evaluate the function at the right endpoints of the subintervals.

(b)(5 points) Find a Riemann integrable function $f(x)$ on the interval $[0, 1]$ such that the formula for the Riemann sum from (a) equals the following formula,

$$\sum_{k=1}^n \frac{k}{k^2 + n^2}.$$

Show all work.

Name: _____

Problem 4, continued

(c)(10 points) For the function $f(x)$ from (b), use the Fundamental Theorem of Calculus to evaluate the Riemann integral,

$$\int_0^1 f(x)dx.$$

Show all work.

(d)(5 points) Evaluate the limit,

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{k}{k^2 + n^2}.$$

Show all work.

Name: _____

Problem 5: _____ /25

Problem 5(25 points) The hourglass shown is obtained by revolving about the y -axis the region in the first and fourth quadrants of the xy -plane bounded by the y -axis, the line $y = +3$, the line $y = -3$ and the hyperbolic segment,

$$5x^2 - 2y^2 = 2.$$

This region is contained in the rectangle with vertices $(0, +3)$, $(2, +3)$, $(2, -3)$ and $(0, -3)$. Therefore the hourglass is contained in a cylinder of base radius 2 and height 6. What fraction of the volume of the cylinder is occupied by the hourglass? Show all work.

Name: _____

Problem 5, continued

Name: _____

Problem 6: _____ /10

Problem 6(10 points) Evaluate the following antiderivative,

$$\int \frac{x+1}{\sqrt{x^2-4x}} dx.$$

At some point, it will be useful to make an inverse substitution. It will be simplest to use an inverse hyperbolic substitution. You are free to use the following identities,

$$\cosh^2(t) - \sinh^2(t) = 1,$$

$$\frac{d}{dt} \sinh(t) = \cosh(t), \quad \frac{d}{dt} \cosh(t) = \sinh(t).$$

Show all work.

Name: _____

Problem 7: _____ /25

Problem 7(25 points) Evaluate each of the following limits. If the limit diverges, write “limit diverges”. Show all work.

(a)(10 points)

$$\lim_{x \rightarrow 0^+} (1 - \cos(\sqrt{x}))^{\sin(x)}.$$

(b)(5 points)

$$\lim_{x \rightarrow 0^+} \frac{x^3 + 2x^2 + x}{x^5 - 1}.$$

Name: _____

Problem 7, continued

(c)(10 points)

$$\lim_{x \rightarrow 0^+} \frac{\sinh(x)}{\sin(x)}.$$

Name: _____

Problem 8: _____ /25

Problem 8(25 points) Evaluate the following improper integrals. If the integral diverges, write “integral diverges”. Show all work.

(a)(10 points)

$$\int_{0^+}^1 \frac{1}{x^s} dx, \quad s > 1.$$

(b)(10 points)

$$\int_{0^+}^1 \frac{1}{x^s} dx, \quad 0 < s < 1.$$

Name: _____

Problem 8, continued

(c)(5 points)

$$\int_1^{\infty} te^{-t^2} dt.$$

Name: _____

Problem 9: _____ /25

Problem 9(25 points) Compute the Taylor series expansion of the following function about $x = 0$,

$$f(x) = \cosh(x) = \frac{1}{2}(e^x + e^{-x}).$$

Show all work.

Extra credit(5 points) Determine the radius of convergence of the power series. Say what test you are using, and show all work.

Name: _____

Problem 10: _____ /25

Problem 10(25 points) Evaluate each of the following integrals. Show all your work.

(a)(5 points)

$$\int_1^2 \frac{dt}{t^{2/3}(1+t^{2/3})}.$$

(b)(5 points)

$$\int_0^{\pi/2} \tan^2(\theta)d\theta.$$

Hint: What identity do you know for $\tan^2(\theta)$?

Name: _____

Problem 10, continued

(c)(5 points)

$$\int_{-2}^0 \frac{dx}{x^2 + 4x + 8}$$

(d)(5 points)

$$\int_{-10e^\pi}^{+10e^\pi} \frac{x}{\sqrt{1+x^{10}}} dx.$$

Name: _____

Problem 10, continued

(e)(5 points)

$$\int_1^2 x[\ln(x)]^2 dx.$$