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18.01 Single Variable Calculus
Fall 2006

18.01 Practice Questions for Exam 2

Solutions will be posted on the 18.01 website. No books, notes, calculators. Show work.

1. For the function $3x^5 - 5x^3 + 1$, sketch the graph over a suitable interval showing all the local maximum and minimum points on the graph, the points of inflection, and the approximate location of its zeros (show on which intervals of the form $[n, n + 1]$, (n is an integer) they occur. Show work, or indicate reasoning.

2. Sketch the graph of $4x^2 - \dfrac{1}{x}$ over an interval showing its interesting features – local maxima and minima, points of inflection, zeros, asymptotes.

3. A line of negative slope through $(1, 2)$ cuts off a triangle in the first quadrant. For which such line will the triangle have least area? (Use its slope m as the independent variable. Show that you get a minimum.)

4. The bottom of the legs of a three-legged table are the vertices of an isosceles triangle with sides 5, 5, and 6. The legs are to be braced at the bottom by three wires in the shape of a Y. What is the minimum length of wire needed? Show it is a minimum.

5. A 200 foot tree is falling in the forest; the sun is directly overhead. At the moment when the tree makes an angle of 30° with the horizontal, its shadow is lengthening at the rate of 50 feet/sec. How fast is the angle changing at that moment?

6. A container in the shape of a right circular cone with vertex angle a right angle is partially filled with water.

   a) Suppose water is added at the rate of 3 cu.cm./sec. How fast is the water level rising when the height $h = 2$cm.?

   b) Suppose instead no water is added, but water is being lost by evaporation. Show the level falls at a constant rate. (You will have to make a reasonable physical assumption about the rate of water loss—state it clearly.)

7. How should the parameter λ be chosen so that $f(x) = \dfrac{e^{-\lambda x}}{1 + 2\sin x}$ remain as close to 1 as possible, when $x \approx 0$? Using this value of λ, estimate $f(.1)$ to two decimal places.

8. State the Mean-value Theorem, and use it to prove that

   a) if $f(x)$ is differentiable and $f'(x) > 0$ for all x, then $f(x)$ is an increasing function;

   b) $e^x > 1 + x$ for all $x > 0$.

9. Evaluate: $\quad a)\ \displaystyle\int \frac{dx}{(3x + 2)^2} \qquad b)\ \displaystyle\int \sin 2x \sin x \, dx \qquad c)\ \displaystyle\int \frac{\ln^2 x}{x} \, dx$

10. Find the function $y(x)$ satisfying $\dfrac{dy}{dx} = xy^2 + x, \quad y(0) = 1$.

11. The rate at which a body heats up by conduction is proportional to the difference between its temperature T and the temperature T_e of its surroundings. A fish at room temperature (20°) is cooked by putting it into boiling water at 100° . After 5 minutes its temperature has risen to 30°. How long will it take to be done (60°) ?

   a) Set up a differential equation for the fish temperature $T(t)$. (Call the constant of proportionality k.)

   b) Find the solution $T(t)$ satisfying the given data, then use it to answer the question.