ATENÇÃO: **Não é permitido destacar as folhas**

$4^{\underline{a}}$ Prova de MA111 — Turmas A e B — 27/06/2014

NOME: GABARITO **RA:**

ASSINATURA:

1. (2,0 pontos) Calcule a área da região limitada pelo eixo x, pelas retas $x = 2$ e $x = 3$ e pela curva

$$y = \frac{x}{x^2 - 5x + 4}$$

2. (1,5 pontos) Determine a solução $y = y(x)$ da equação diferencial

$$\frac{dy}{dx} = \frac{\operatorname{arcsen} x}{y^2(1-x^2)^{1/2}}$$

que satisfaz a condição inicial $y(0) = 0$.

3. Calcule:

$$(\mathbf{a})(1,5)\ \int_{-\infty}^{0} (x+3)e^{2x}dx \qquad (\mathbf{b})(1,5)\ \int_{1}^{3} \frac{|x-2|}{x}dx$$

(c) (1,5) Calcule o valor da constante c para que

$$\lim_{x\to 0} \frac{\int_0^x \frac{t^2\,dt}{\sqrt{c+t}}}{x - \operatorname{sen} x} = 1$$

4. Determine o volume do sólido gerado pela rotação da região limitada por $y^2 = 8x$ e $x = 2$:

(a) (1,5 pontos) em torno da reta $y = -4$

(b) (1,5 pontos) em torno da reta $x = 3$

1) $f(x) = y = \dfrac{x}{x^2 - 5x + 4} = \dfrac{x}{(x-4)(x-1)}$

		0		1		4	
x	$-$		$+$		$+$		$+$
$x-4$	$-$		$-$		$-$		$+$
$x-1$	$-$		$-$		$+$		$+$
$f(x)$	$-$		$+$		$-$		$+$

$\Rightarrow$ (*) $\begin{cases} f(x) < 0 & \text{se} \\ 1 < x < 4 \end{cases}$

$$\text{Área} \overset{(*)}{=} -\int_2^3 \frac{x\,dx}{(x-4)(x-1)}$$

0,5

$$\frac{x}{(x-4)(x-1)} = \frac{A}{x-4} + \frac{B}{x-1} = \frac{4/3}{x-4} + \frac{-1/3}{x-1}$$

$$x = A(x-1) + B(x-4) \Rightarrow \begin{cases} x = 1 \Rightarrow 1 = -3B \Rightarrow B = -1/3 \\ x = 4 \Rightarrow 4 = 3A \Rightarrow A = 4/3 \end{cases}$$

$$\text{Área} = -\left(\frac{4}{3}\int_2^3 \frac{dx}{x-4} - \frac{1}{3}\int_2^3 \frac{dx}{x-1} \right)$$

$$= \left(-\frac{4}{3}\ln|x-4| + \frac{1}{3}\ln|x-1| \right) \Bigg|_{x=2}^{x=3}$$

$$= \left(-\frac{4}{3}\overset{=0}{\ln 1} + \frac{1}{3}\ln 2 \right) - \left(-\frac{4}{3}\ln 2 + \frac{1}{3}\overset{=0}{\ln 1} \right)$$

$$= \boxed{\frac{5}{3}\ln 2}$$

1,0

2) $$\frac{dy}{dx} = \frac{\text{arc sen } x}{y^2(1-x^2)^{1/2}} \quad , \quad y(0) = 0$$

$$y^2\,dy = \frac{\text{arc sen } x}{\sqrt{1-x^2}}$$

$$\int y^2\,dy = \frac{y^3}{3} + C_1$$

$$\begin{cases} u = \text{arc sen } x \\ du = \dfrac{dx}{\sqrt{1-x^2}} \end{cases}$$

$$\int \frac{\text{arc sen } x \;\; dx}{\sqrt{1-x^2}} = \int u\,du = \frac{u^2}{2} + C_2$$

$$= \frac{1}{2}(\text{arc sen } x)^2 + C_2$$

<u>SOLUÇÃO GERAL DA EQUAÇÃO DADA:</u>

$$\frac{y^3}{3} = \frac{1}{2}(\text{arc sen } x)^2 + C$$

$$y(0) = 0 \implies \frac{0}{3} = \frac{1}{2}(\text{arc sen } 0)^2 + C = C$$

$$\implies \boxed{C = 0}$$

<u>RESPOSTA:</u> $\boxed{y^3 = \frac{3}{2}(\text{arc sen } x)^2}$

$\underline{\underline{3(a)}}$:

$$\int \underbrace{(x+3)}_{u}\, \underbrace{e^{2x}\,dx}_{dv} = uv - \int v\,du \qquad \begin{cases} u = x+3 \\ du = dx \end{cases}$$

$$= (x+3)\frac{e^{2x}}{2} - \int \frac{e^{2x}}{2}\,dx \qquad \begin{cases} dv = e^{2x}\,dx \\ v = \dfrac{e^{2x}}{2} \end{cases}$$

$$= (x+3)\frac{e^{2x}}{2} - \frac{e^{2x}}{4} + C$$

$$= \frac{x}{2}e^{2x} + \frac{5}{4}e^{2x} + C$$

$$\int_{-\infty}^{0} (x+3)e^{2x}\,dx = \lim_{t\to-\infty} \int_{t}^{0} (x+3)e^{2x}\,dx$$

$$= \lim_{t\to-\infty} \left(\frac{x}{2}e^{2x} + \frac{5}{4}e^{2x}\right)\Bigg|_{x=t}^{x=0}$$

$$= \lim_{t\to-\infty} \left[\frac{5}{4} - \left(\frac{5}{4} + \frac{t}{2}\right)e^{2t}\right]$$

$$= \frac{5}{4} - \lim_{t\to-\infty} \frac{\frac{5}{4} + \frac{t}{2}}{e^{-2t}}$$

$$\overset{L'H}{=} \frac{5}{4} - \lim_{t\to-\infty} \frac{1/2}{-2e^{-2t}}$$

$$= \frac{5}{4} - \frac{1/2}{-\infty} = \frac{5}{4} - 0 = \frac{5}{4} \;//$$

3(b): $\displaystyle\int_1^3 \frac{|x-2|}{x}\,dx$

$$= \int_1^2 \frac{|x-2|}{x}\,dx + \int_2^3 \frac{|x-2|}{x}\,dx$$

$$= \int_1^2 \frac{-x+2}{x}\,dx + \int_2^3 \frac{x-2}{x}\,dx$$

0,7

$$= \left(-x + 2\ln|x|\right)\Big|_{x=1}^{x=2} + \left(x - 2\ln|x|\right)\Big|_{x=2}^{x=3}$$

$$= \left[\left(-2+2\ln 2\right) - \left(-1+2\ln 1\right)\right] + \left[\left(3-2\ln 3\right) - \left(2-2\ln 2\right)\right]$$

$$= 4\ln 2 - 2\ln 3 = 2\ln\frac{4}{3} = \ln\frac{16}{9}.$$

0,8

3(c):

$$1 = \lim_{x\to 0} \frac{\displaystyle\int_0^x \frac{t^2\,dt}{\sqrt{c+t}}}{x - \operatorname{sen} x} \quad \left(= \frac{0}{0}\right)$$

$$\overset{L'H}{=} \lim_{x\to 0} \frac{\dfrac{x^2}{(c+x)^{1/2}}}{1-\cos x}$$

$$= \lim_{x\to 0} \frac{x^2}{(c+x)^{1/2}\,(1-\cos x)} \quad \left(= \frac{0}{0}\right)$$

$$\overset{L'H}{=} \lim_{x\to 0} \frac{2x}{\frac{1}{2}(c+x)^{-1/2}(1-\cos x) + (c+x)^{1/2}\operatorname{sen} x} \quad \left(= \frac{0}{0}\right)$$

$$\overset{L'H}{=} \lim_{x\to 0} \frac{2}{-\frac{1}{4}(c+x)^{-3/2}(1-\cos x) + \frac{1}{2}(c+x)^{-1/2}\operatorname{sen} x + \frac{1}{2}(c+x)^{-1/2}\operatorname{sen} x + (c+x)^{1/2}\cos x}$$

$$= \frac{2}{c^{1/2}}$$

$$\Longrightarrow \quad c^{1/2} = 2 \Longrightarrow \boxed{c = 4}$$

3(c): OUTRA RESOLUÇÃO

$$\begin{cases} u = c+t \\ du = dt \end{cases}$$

$$\int \frac{t^2\,dt}{\sqrt{c+t}} = \int \frac{(u-c)^2\,du}{u^{1/2}}$$

$$= \int \left(u^{3/2} - 2c\,u^{1/2} + c^2 u^{-1/2} \right) du = \frac{2}{5}u^{5/2} - \frac{4}{3}c\,u^{3/2} + 2c^2 u^{1/2} + C$$

$$= \frac{2}{5}(c+t)^{5/2} - \frac{4c}{3}(c+t)^{3/2} + 2c^2(c+t)^{1/2} + C$$

$$\int_0^x \frac{t^2\,dt}{\sqrt{c+t}} = \frac{2}{5}(c+x)^{5/2} - \frac{4c}{3}(c+x)^{3/2} + 2c^2(c+x)^{1/2} - \frac{16}{15}c^{5/2}$$

0,7

$$1 = \lim_{x\to 0} \frac{\int_0^x \frac{t^2\,dt}{\sqrt{c+t}}}{x - \operatorname{sen} x}$$

$$= \lim_{x\to 0} \frac{\frac{2}{5}(c+x)^{5/2} - \frac{4c}{3}(c+x)^{3/2} + 2c^2(c+x)^{1/2} - \frac{16}{15}c^{5/2}}{x - \operatorname{sen} x}$$

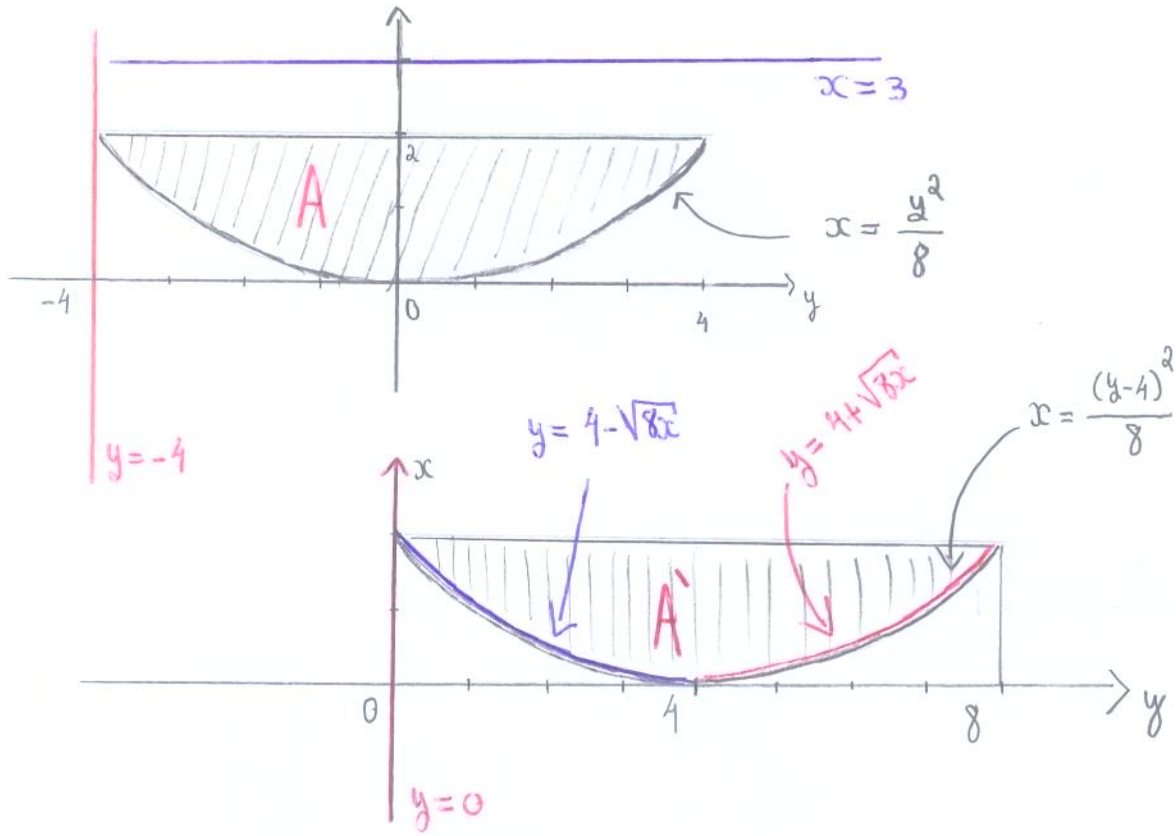
$$\overset{L'H}{=} \lim_{x\to 0} \frac{(c+x)^{3/2} - 2c(c+x)^{1/2} + c^2(c+x)^{-1/2}}{1 - \cos x} =$$

$$\overset{L'H}{=} \lim_{x\to 0} \frac{\frac{3}{2}(c+x)^{1/2} - c(x+c)^{-1/2} - \frac{c^2}{2}(x+c)^{-3/2}}{\operatorname{sen} x}$$

$$\overset{L'H}{=} \lim_{x\to 0} \frac{\frac{3}{4}(c+x)^{-1/2} + \frac{c}{2}(x+c)^{-3/2} + \frac{3}{4}c^2(x+c)^{-5/2}}{\cos x} = 2c^{-1/2}$$

0,8 $\Longrightarrow$ $\boxed{c = 4}$

4(a):



V_1 = Volume do sólido gerado pela rotação da região A em torno da reta $y=-4$
= Volume do sólido gerado pela rotação da região A` em torno do eixo x

$$V_1 = 2\pi \int_0^8 y \cdot 2\, dy - 2\pi \int_0^8 y\, \frac{(y-4)^2}{8}\, dy = 2\pi \int_0^8 y \left(2 - \frac{(y-4)^2}{8}\right) dy$$

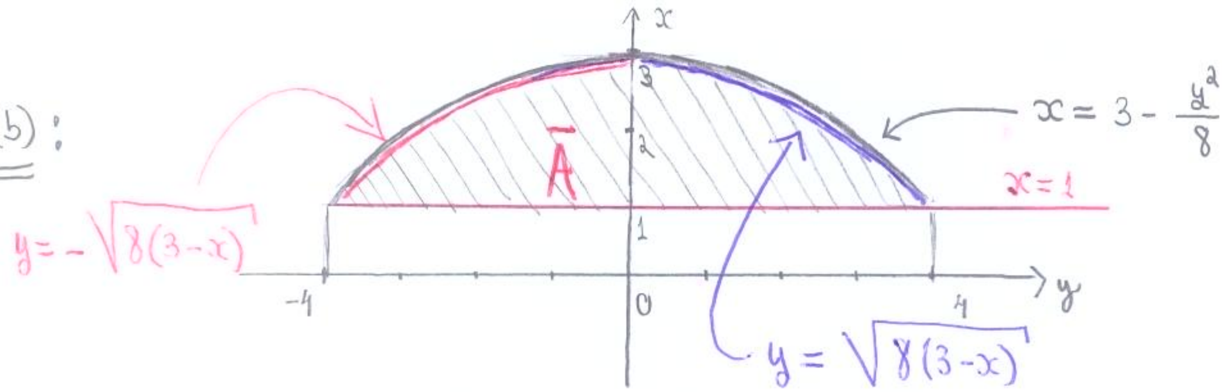
$$= 2\pi \int_0^8 y\left(-\frac{y^2}{8} + y\right) dy = 2\pi \int_0^8 \left(-\frac{y^3}{8} + y^2\right) dy$$

$$= 2\pi \left(-\frac{y^4}{32} + \frac{y^3}{3}\right)\Bigg|_{y=0}^{y=8} = \frac{256}{3}\pi$$

OUTRA FORMA:

$$V_1 = \pi \int_0^2 \left(4+\sqrt{8x}\right)^2 dx - \pi \int_0^2 \left(4-\sqrt{8x}\right)^2 dx$$

4(b):



V_2 = Volume do sólido gerado pela rotação da região A em torno da reta $x = 3$

= Volume do sólido gerado pela rotação da região $\bar{A}$ em torno do eixo y

$$V_2 = \pi \int_{-4}^{4} \left(3 - \frac{y^2}{8}\right)^2 dy - \pi \int_{-4}^{4} 1^2 dy = \pi \int_{-4}^{4} \left[\left(3 - \frac{y^2}{8}\right)^2 - 1\right] dy$$

$$= \pi \int_{-4}^{4} \left(8 - \frac{3}{4}y^2 + \frac{y^4}{64}\right) dy = \pi \left(8y - \frac{y^3}{4} + \frac{y^5}{5 \times 64}\right) \Bigg|_{y=-4}^{y=4}$$

$$= 2\pi \frac{96}{5} = \frac{192}{5}\pi$$

OUTRA FORMA:

$$V_2 = 2 \cdot \left(2\pi \int_{1}^{3} x\sqrt{8(3-x)}\, dx\right)$$

$$\begin{cases} u = 3 - x \\ du = -dx \\ x = 1 \Rightarrow u = 2 \\ x = 3 \Rightarrow u = 0 \end{cases}$$

$$= 2 \cdot \left(2\pi \int_{2}^{0} (3-u)\sqrt{8u}\, (-du)\right)$$

$$= 2 \cdot \left(2\pi \int_{0}^{2} (3-u)\sqrt{8u}\, du\right)$$

$$= 2 \cdot \frac{96}{5}\pi = \frac{192}{5}\pi$$